Jour. Ind. Soc. Ag. Statistics
Vol. XLI, No. 2 (1989), pp. 173-175

ON THE ESTIMATION OF MEAN WHEN POPULATION VARIANCE IS KNOWN

POONAM KATARIA and SUKHMINDER SINGH
Punjab Agricultural University, Ludhiana-141004

(Received : November, 1987)

SUMMARY

A class of estimators for population mean when the population variance is known has been suggested. It is shown that the optimum estimator in the class has a smallar bias as compared to estimators proposed earlier.

Keywords : Optimum estimator, mean square error, large sample approximations.

Introduction

Let $y_1, y_2, \ldots, y_n$ be a random sample of size n from a population with unknown mean $\bar{Y}$ and known variance σ^2. Upadhyaya and Srivastava [2] proposed the estimator

$$\bar{Y}_1 = \bar{y} + \sigma^2/n\bar{y} \tag{1.1}$$

and obtained the bias and mean squared error (MSE) to order $O(n^{-2})$ as

$$\text{Bias}\ (\bar{Y}_1) = (\sigma^2/n\bar{Y})\ (1 + \sigma^2/n\bar{Y}^2) \tag{1.2}$$

and $\text{MSE}\,(\hat{y}_1) = (\sigma^2/n)\,(1 - \sigma^2/n\bar{Y}^2)$ (1.3)

Upadhyaya and Singh [1] proposed an equally efficient estimator

$$\hat{y}_2 = \bar{y} + \sigma^2\ \bar{Y}/n\bar{y}(^2 + \sigma^2) \tag{1.4}$$

but with smaller bias to order $O\,(n^{-2})$ as

$$\text{Bias}\,(\hat{y}_2) = \sigma^2/n\bar{Y} \tag{1.5}$$

The purpose of this paper is to consider a class of estimators of the mean when population variance is known. The large sample approximations have also been investigated.

2. Proposed Class of Estimators

Assuming the population variance σ^2 to be known, we consider the class of estimators for the mean $\bar{Y}$ as

$$\hat{y}_3 = \bar{y} + K\sigma^2\,\bar{y}\delta - \sigma^2/n\bar{y} \tag{2.1}$$

where $\delta = (n\bar{y}^2 + \sigma^2)^{-1}$ and K is the characterizing scalar to be chosen suitably.

The bias, MSE and $K_{\text{opt.}}$ for minimum MSE are

$$\text{Bias}\,(\hat{y}_3) = K\sigma^2\,E\,(\bar{y}\delta) - \sigma^2\,E\,(n\bar{y})^{-1} \tag{2.2}$$

$$\begin{aligned}\text{MSE}\,(\hat{y}_3) = {} & E\,(\hat{y}_3 - \bar{Y})^2 + K^2\,\sigma^4\,E\,(\bar{y}\,\delta)^2 + \sigma^4\,E\,(n^2\,\bar{y}^2)^{-1} \\ & + 2K\,\sigma^2\,E\,(\bar{Y}^2\delta) - 2\,K\,\sigma^2\,\bar{Y}\,E\,(\bar{y}\delta) \\ & - 2\,K\sigma^4\,n^{-1}\,E(\delta) + 2\,\sigma^2\,\bar{Y}\,E\,(n\bar{y})^{-1} - 2\,\sigma^2\,n^{-1}\end{aligned} \tag{2.3}$$

and $K_{\text{opt.}} = [\bar{Y}\,E\,(\bar{y}\delta) + \sigma^2\,n^{-1}\,E(\delta) - E\bar{y}^2\delta)]\,/\,[\sigma^2\,E(\bar{y}\delta)^2]$ (2.4)

The expectations involved in these expressions are mathematically intractable. We, therefore, resort to derive the large sample approximations.

3. Large Sample Approximations

We write

$$\bar{y} = \bar{Y} + \varepsilon$$

where ε is of the order $O(n^{-1/2})$ with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma^2/n$. Choosing n large enough so that $|\varepsilon / \bar{Y}| < 1$, we may easily get the expectations by retaining terms of order $O(n^{-2})$ as

$$E(n\bar{y})^{-1} = (1 + \sigma^2/n\bar{Y}^2)/n\bar{Y},\ E(n^2\bar{y}^2)^{-1} = (n^2\bar{Y}^2)^{-1}$$

$$E(\delta) = (1 + 2\sigma^2/n\bar{Y}^2)/n\bar{Y}^2,\ E(\bar{y}\delta) = (n\bar{Y})^{-1} \quad (3.1)$$

and $E(\bar{y}\delta)^2 = (n^2\bar{Y}^2)^{-1}$ and $E(\bar{y}^2\delta) = 1 - \sigma^2/n\bar{Y}^2)/n$

From (2.2), (2.3), (2.4) and (3.1) we get (upto terms of order $O(n^{-2})$)

$$\text{Bias}(\bar{y}_3) = \sigma^2 (K - 1 - \sigma^2/n\bar{Y}^2)/n\bar{Y} \quad (3.2)$$

$$\text{MSE}(\bar{y}_3) = \sigma^2 [1 + (K - 1)(K - 3)\sigma^2/n\bar{Y}^2]/n \quad (3.3)$$

and $K_{opt.} = 2.$ (3.4)

4. Comparison

For $K_{opt.}$, the estimator $\bar{y}_3$, its bias and MSE become

$$\bar{y}_3 = \bar{y} + \sigma^2\bar{y}(1 - \sigma^2/n\bar{y}^2)/(n\bar{y}^2 + \sigma^2) \quad (4.1)$$

$$\text{Bias}(\bar{y}_3) = (\sigma^2/n\bar{Y})(1 - \sigma^2/n\bar{Y}^2) \quad (4.2)$$

and $\text{MES}(\bar{y}_3) = (\sigma^2/n)(1 - \sigma^2/n\bar{Y}^2)$ (4.3)

From (1.2), (1.5) and (4.2), we have Bias $(\bar{Y}_3) <$ Bias $(\bar{Y}_2) <$ Bias $(\bar{Y}_1)$ upto $O(n^{-2})$. And all three estimators have equal MSE. Therefore, the estimator $\bar{y}_3$ is an improvement over $\bar{y}_2$ and $\bar{y}_1$.

REFEREECES

[1] Upadhyaya, L. N. and Singh, H, P. (1984) : Estimation of mean when population variance is known. *Jour. Ind. Soc. Agri. Stat.* 36 : 19-22.

[2] Upadhyaya, L. N. and Srivastava, S. R. (1976) : An efficient estimator of mean when population variance is known. *Jour. Ind. Soc. Agri. Stat.* 28 : 99-102.